Dynamic Compression Characteristics of Flexible Foams. I. Similarity Model, Analysis, and Experiments

S. YOSSIFON* and M. SZANTO, *N.R.C.N., P.O. Box 9001, Beer-Sheva, Israel*

Synopsis

A simple mathematical model is proposed, based on dimensional similarity parameters, to describe the characteristics of flexible plastic foams under impact conditions. The model assumes that the foam is rate-dependent material, when the dynamic stress is a function of the strain and the strain rate. The similarity parameters include the geometric dimension of the foam, the mass of the absorbing body, and the drop height. By using this model, one can predict the maximum deformation, the maximum decelerations, and the time-pulse period for a wide range of drop heights and masses, by conducting several drop tests. We verified the efficacy of this model by performing free-fall drop tests with flexible polyurethane foam having a uniform density of 240 kg/m^3.

INTRODUCTION

Recently, significant expansion has taken place in the use of plastic foams as protective packaging materials. Much research has gone into studying the cushioning properties of these foams for static and dynamic purposes. The flexible foams, such as polyurethane flexible foams, are commonly used for shock absorbing to protect goods against impact or against any damage caused by dynamic action. Whereas rigid foams can be used for one impact test only, the flexible foams are intended for multiple uses and for long lifetimes.

The designer of the protective packaging should choose the proper foam for specific needs. This means choosing the shape and the density of the foam for absorbing the shock to a known mass, while ensuring maximum permissible deceleration and deformation.

The essential facts that are needed to define an impact condition are: the maximum deceleration of the mass of the body, the maximum deformation of the foam (i.e., maximum displacement) and the time-pulse period of the impact. This information enables the designer to estimate the resistibility of the body under impact conditions.

Recently, two terms—ideality and efficiency—were proposed to describe the foams.[1] These parameters can be predicted from static load–deformation experiments and dynamic tests. The use of these parameters makes possible a more precise definition of the properties of the foam. However, as yet, there are no explicit mathematical formulations which can be used to define the dynamic responses of the foam.

*To whom correspondence should be addressed.

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Shuttleworth et al.[2] proposed a filament buckling model to explain the form of the observed stress-strain relationship. They predicted an approximate stress–strain equation which was obtained by experimental relationships which were proposed[3] without a physical basis for explanation.

The goal of this work is to propose a simple mathematical model to predict the maximum deceleration, the maximum deformation, and the time-pulse period under drop-test conditions for flexible plastic foams. With this approach, only a few experiments would be needed to plot curves of dimensional similarity parameters for a wide range of geometrical and test conditions for the same density of foam.

THEORY

A body of mass M is dropped from height H on foam of uniform density ρ, thickness h, and cross-sectional area A. The force F that exerts mass M on the foam at the time of impact is

$$F = M\frac{d^2y}{dt^2} + Mg \tag{1}$$

where y is a displacement of the contact surface between the foam and the mass and g is a gravity acceleration. We assume a constant foam cross-sectional area at impact. This assumption has been proved experimentally, as being quite true for a wide range of drop tests.

We introduce the displacement–strain relationship and its derivations as

$$\begin{aligned} y &= h \cdot \epsilon \\ dy/dt &= h\,d\epsilon/dt \\ d^2y/dt^2 &= h\,d^2\epsilon/dt^2 \end{aligned} \tag{2}$$

where ϵ is the strain in the foam.

Combining eq. (2) with eq. (1) gives

$$\sigma(\epsilon, d\epsilon/dt) = (\sigma_{st} \cdot h)(d^2\epsilon/dt^2/g) + \sigma_{st} \tag{3}$$

where the static stress σ_{st} is

$$\sigma_{st} = M \cdot g/A$$

and the normal dynamic compression stress σ is a function of the strain and the strain rate.

The initial conditions at the time of impact which satisfy eq. (3) are:

(a) The initial displacement is zero:

$$y|_{t=0} = 0$$

(b) The initial velocity is equal to the free-fall velocity of the mass M at impact time:

$$dy/dt|_{t=0} = \sqrt{2g \cdot H}$$

where H is the drop height.

Actually, in severe impact conditions, the second term of the right member can be ignored in eq. (3), because the dynamic stress to static stress ratio $\sigma(\epsilon, d\epsilon/dt)/\sigma_{st}$, is much higher than unity. Therefore, maintaining the same initial conditions and keeping a constant value for the term $\sigma_{st} \cdot h$ from eq. (3) yields one possible solution only. Mathematically, one can write

$$\sigma(\epsilon, d\epsilon/dt) = C_1 \cdot \left(d^2\epsilon/dt^2/g\right) \tag{4}$$

where $C_1 = \sigma_{st} \cdot h$, with initial conditions:

$$d\epsilon/dt|_{t=0} = C \left(\text{or } C_2 = H/h^2\right)$$

and

$$\epsilon|_{t=0} = C_3\,(C_3 = 0)$$

where C_2 and C_3 are initial parameters.

The similarity parameters are: (a) $\sigma_{st} \cdot h$ and (b) H/h^2; for the same numerical values for similarity parameters, one can expect to obtain the same solution of eq. (4), that is, the same impact response in the drop test for the foam material and density.

As a result, one can assume that for the same numerical values for the similarity parameters in the drop test, the same values may be obtained for maximum deceleration, maximum deformation, and time pulse period for the same flexible foam material and density.

One can apply this model by several drop tests for the same plastic foam material and density, in various masses and drop heights, thus obtaining the maximum deceleration, maximum deformation, and the time pulse period for a wide range of drop tests.

EXPERIMENTAL

Material

A total of five locally manufactured polyurethane flexible foams were tested. Each had a density of 240 kg/m^3, but each had a different thickness. The foams were supplied by courtesy of Caesarea Polymers Ltd. The geometrical dimensions of the foams are shown in Table I.

Apparatus and Procedure

The experimental apparatus, shown schematically in Figure 1, consisted of a changeable mass (of up to 60 kg), guied by eight rollers on a vertical guiding rod, which was 50 mm in diameter and 4.5 m long. The mass is positioned at the desired height H by an electrical winch, and the foam is positioned at the bottom of the rod.

The deceleration pulse is measured by a piezoresistive accelerometer, and the deformation is measured by a photosensor with 0.5-mm resolution. The effective drop height H_{eff} and the maximum height that the mass reached after bouncing off of the foam are calculated from measuring the velocities

TABLE I
Geometrical Dimensions of the Tested Foam

Specimen no.	Cross-section area[a] A (m^2)	Thickness h (m)
1	0.0272	0.1
2	0.0272	0.15
3	0.0272	0.2
4	0.0272	0.25
5	0.0272	0.3

[a] The cross-section geometry is circular.

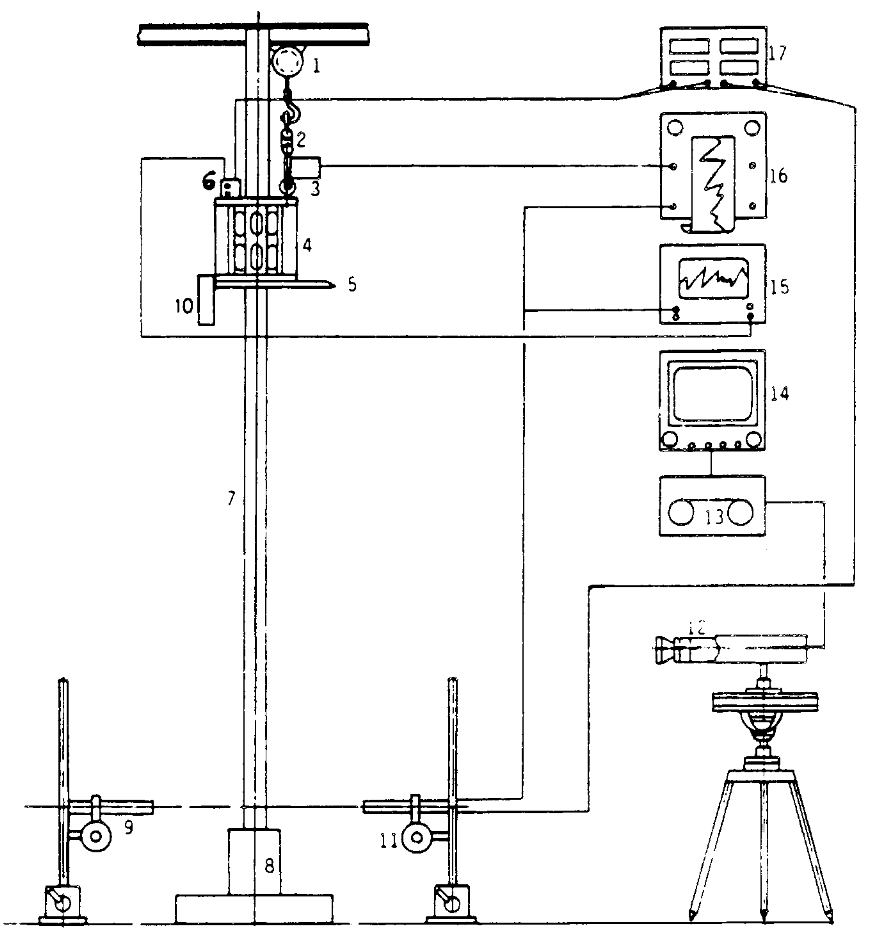

Fig. 1. Experimental setup: (1) electric winch; (2) release mechanism; (3) microswitch; (4) sliding mass; (5) marking knife; (6) Accelerometer; (7) guiding rod; (8) foam; (9) light source; (10) photo-slide; (11) photocell detector; (12) TV camera; (13) videotape; (14) [illegible] (15) storage oscilloscope; (16) X-T recorder; (17) DC power supply.

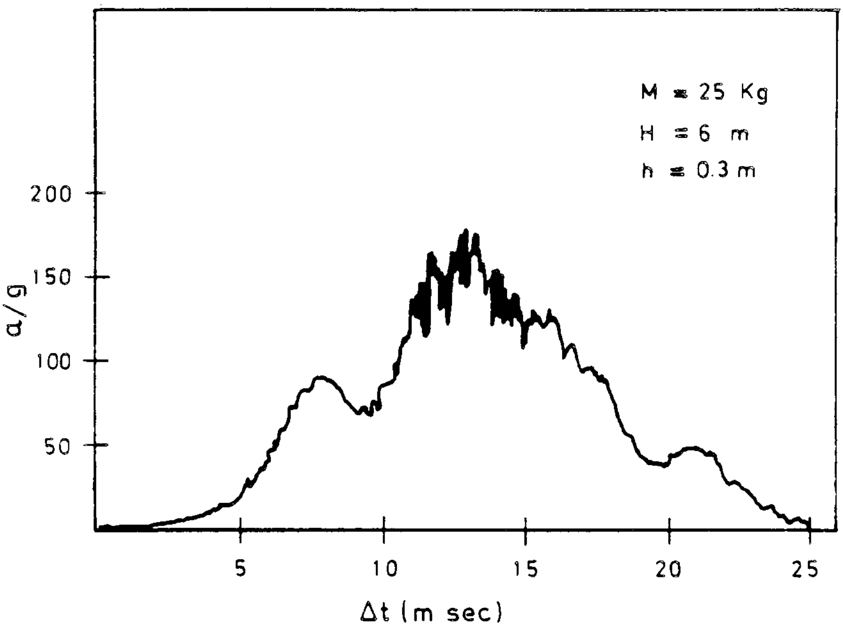

Fig. 2. A typical plot of deceleration vs. time in a drop-test. (The experimental parameters are shown in the figure.)

before and after impact time at $y = 0$ state, to eliminate the friction forces between the mass and the guiders.

The roles of the TV camera in the present experiments were to display the shape of the foam at impact time and, occasionally, to check the foam-mass contact time (in spite of the fact that the impact time was obtained from the output of the photosensor).

The tests were done with different masses and drop heights for each foam (see Table I).

RESULTS

A typical plot of deceleration vs. time is presented in Figure 2. The time duration of the shock pulse Δt is the time of contact, that is, the time elapsing between the initial contact and the separation as the mass bounces upward. Therefore, Δt is obtained only from the results of the photosensor pulses, and we occasionally checked the results by the TV camera. It is difficult to determine precisely the time duration from the deceleration vs. time pulse. It was observed (by TV camera) that the diameter of the foam varied up to 4% maximum at the impact time for all the present drop-tests.

Figures 3–7 show the plots of maximum deceleration $d^2\epsilon/dt^2/g$, maximum deformation ϵ_{max}, and time pulse duration Δt vs. H/h^2 for different drop heights H and with different masses M.

From these plots (Figs. 3–7) a combined plot is predicated and plotted in Figure 8. This combined plot shows a maximum deceleration, a maximum deformation, and a time duration vs. $(\sigma_{st} \cdot h)$ for different H/h^2.

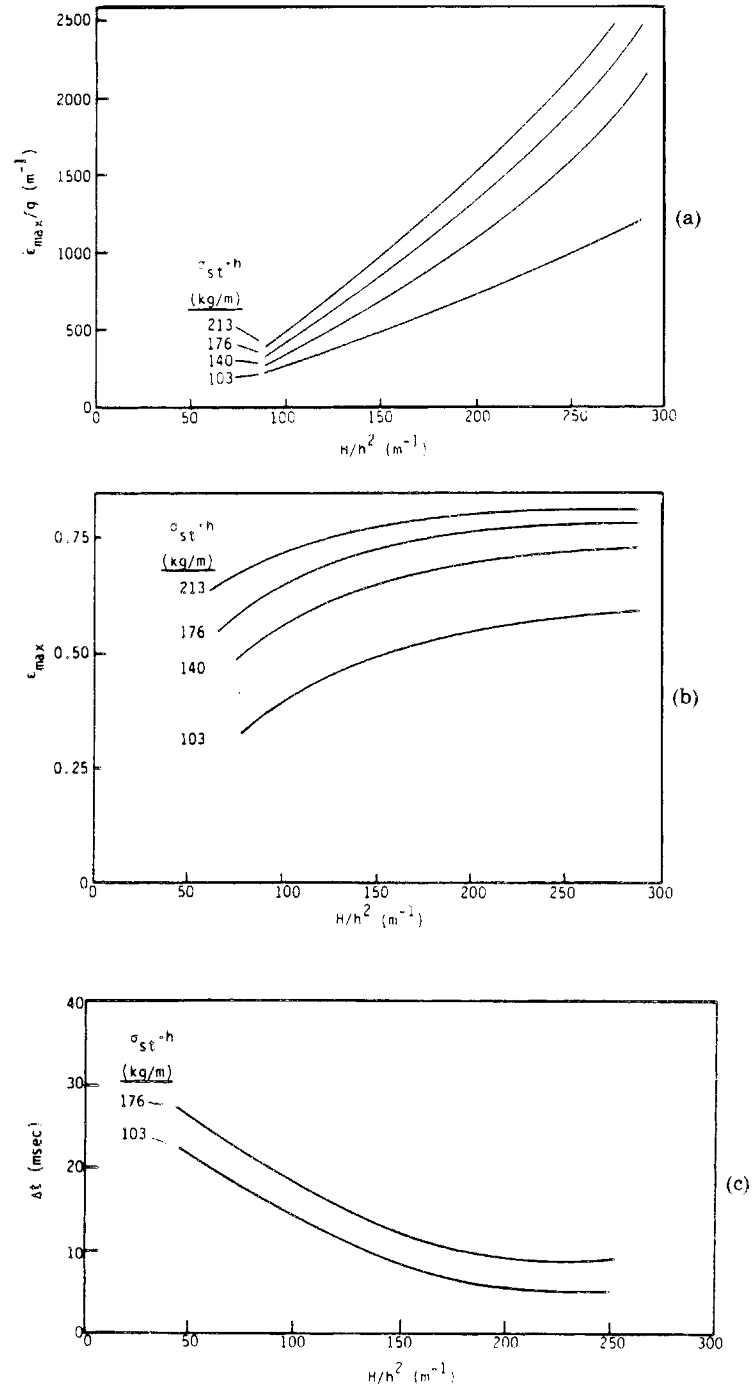

Fig. 3. Plots of (a) Maximum deceleration $(d^2\epsilon/dt^2/g)_{max}$, (b) maximum deformation ϵ_{max} and (c) time pulse period Δt vs. H/h^2 for various values of $\sigma_{st} \cdot h$, for drop test with flexible foam of 0.1-m thickness, 0.0272 m^2 cross-sectional area, and 240 kg/m^3 density.

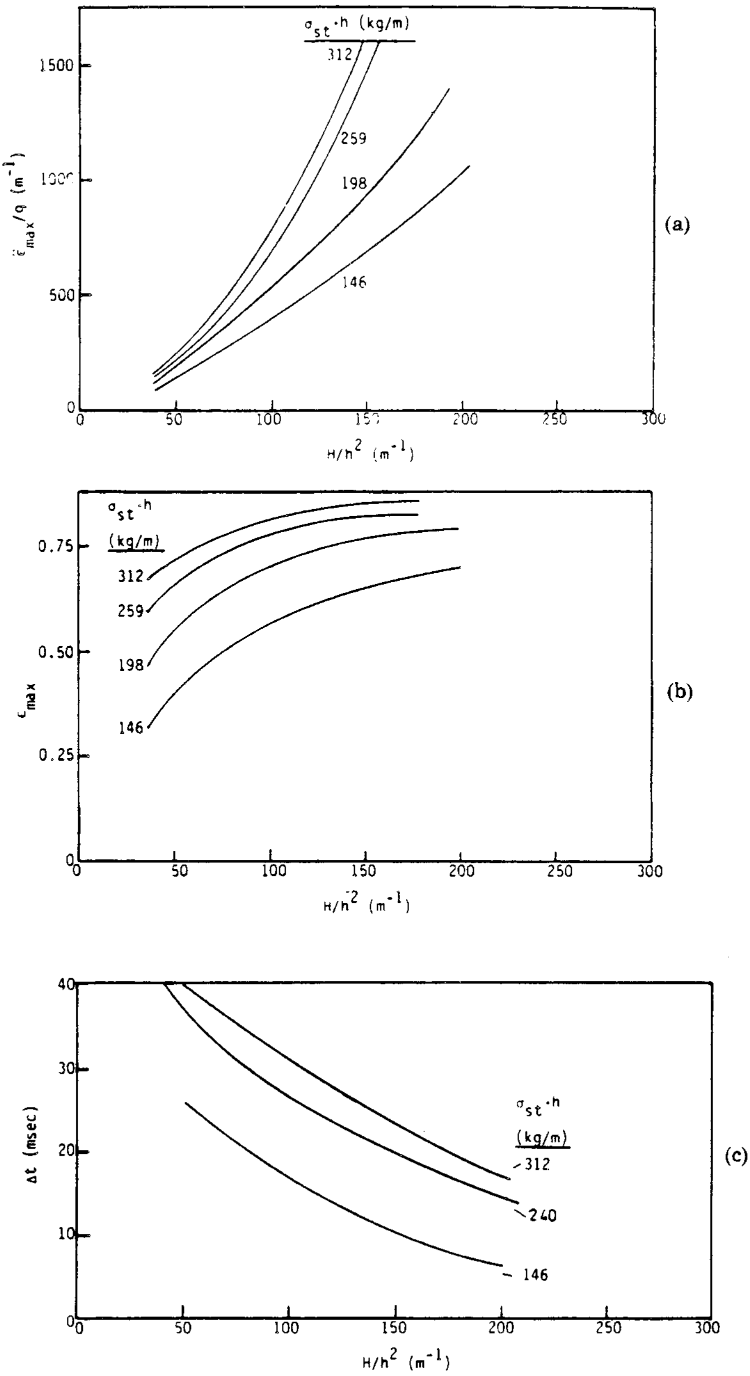

Fig. 4. Plots of (a) Maximum deceleration $(d^2\epsilon/dt^2/g)_{max}$, (b) maximum deformation ϵ_{max}, and (c) time pulse period Δt vs. H/h^2 for various values of $\sigma_{st} \cdot h$, for drop test with flexible foam of 0.15-m thickness, 0.0272 m^2 cross-sectional area, and 240 kg/m^3 density.

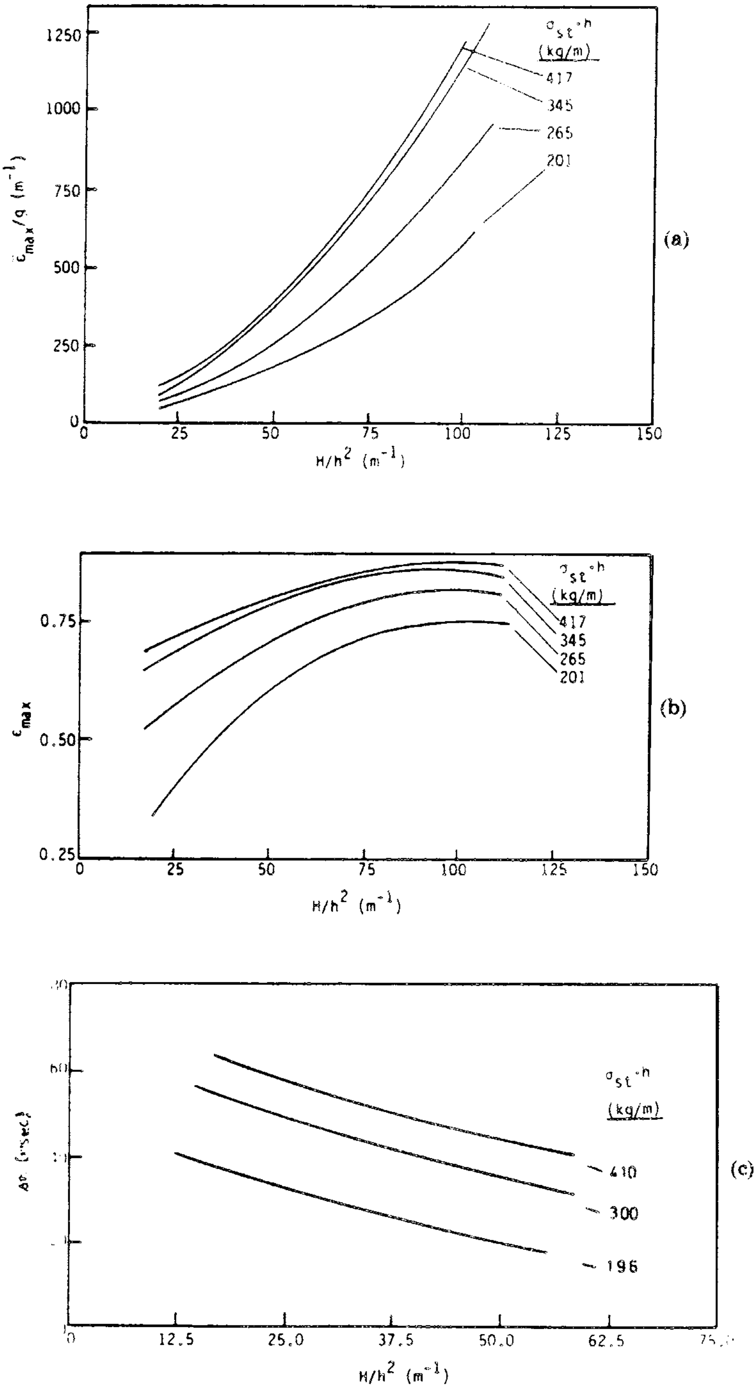

Fig. 5. Plots of (a) Maximum deceleration $(d^2\epsilon/dt^2/g)_{max}$, (b) maximum deformation ϵ_{max}, and (c) time pulse period Δt vs. H/h^2 for various values of $\sigma_{st}\cdot h$, for drop test with flexible foam of 0.2-m thickness, 0.0272 m^2 cross-sectional area, and 240 kg/m^3 density.

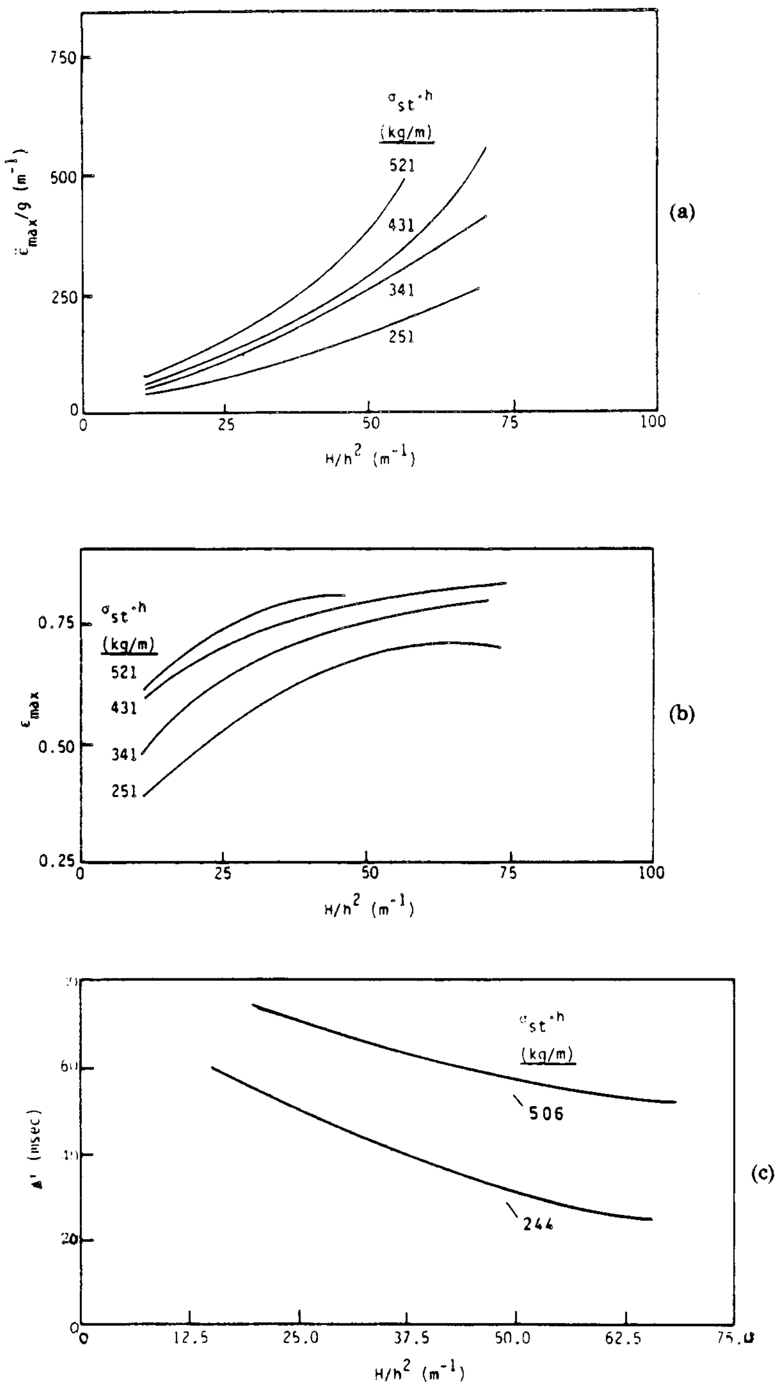

Fig. 6. Plots of (a) Maximum deceleration $(d^2\epsilon/dt^2/g)_{max}$, (b) maximum deformation ϵ_{max}, and (c) time pulse period Δt vs. H/h^2 for various values of $\sigma_{st} \cdot h$, for drop test with flexible foam of 0.25-m thickness, 0.0272 m^2 cross-sectional area, and 240 kg/m^3 density.

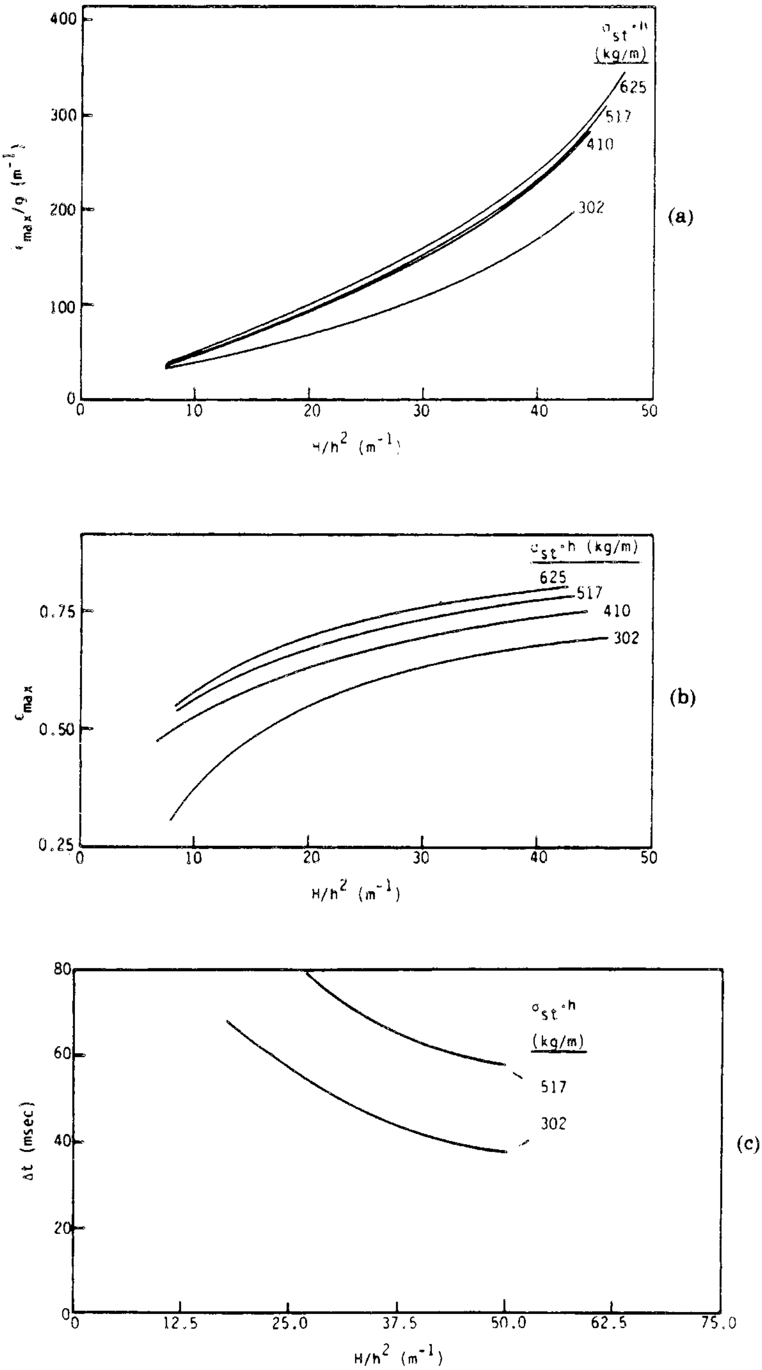

Fig. 7. Plots of (a) Maximum deceleration $(d^2\epsilon/dt^2/g)_{max}$, (b) maximum deformation ϵ_{max}, and (c) time pulse period Δt vs. H/h^2 for various values of $\sigma_{st} \cdot h$, for drop test with flexible foam of 0.3-m thickness, 0.0272 m^2 cross-sectional area, and 240 kg/m^3 density.

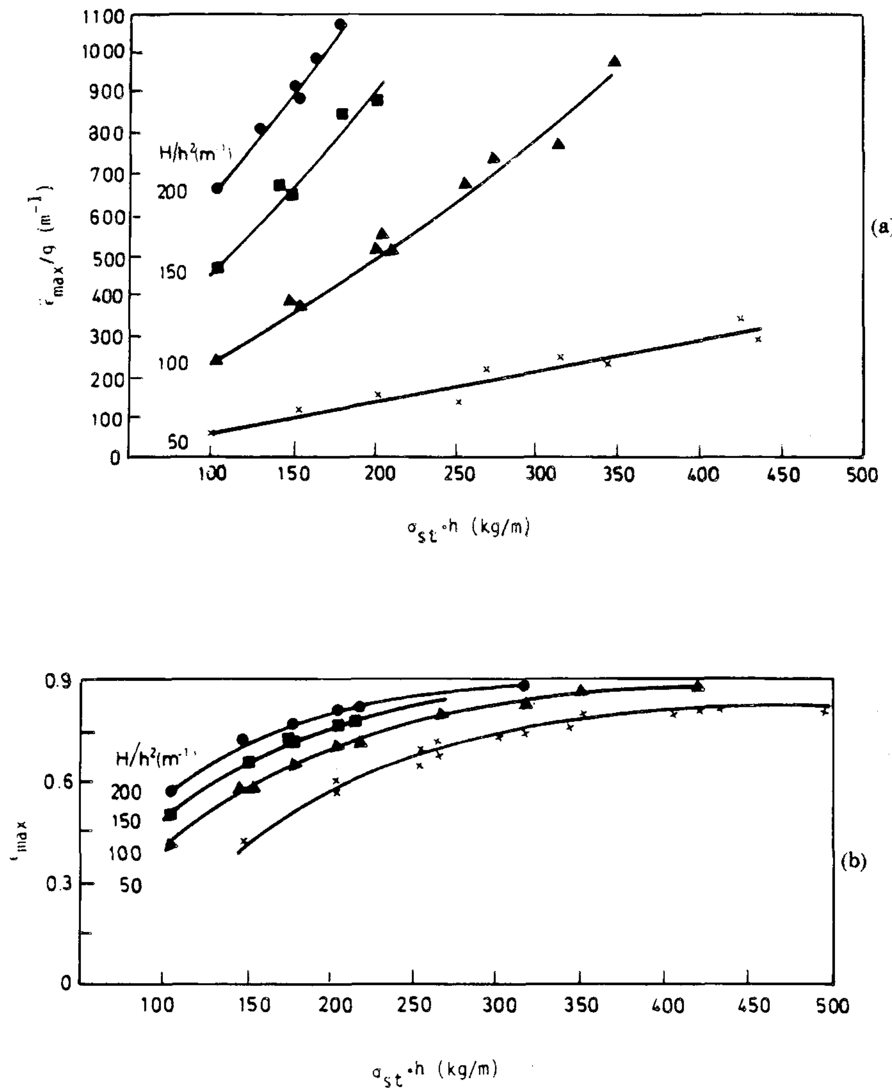

Fig. 8. Combined plot (predicted from Figs. 3–7) of (a) maximum deceleration $d^2\epsilon/dt^2/g$, (b) maximum deformation ϵ_{max}, and (c) time pulse period Δt vs. $\sigma_{st} \cdot h$ for various values of H/h^2: (×) 50; (▲) 100; (■) 150; (●) 200.

Actually, one could reduce the number of the drop tests or could use fewer specimens to predict Figure 8. However, in our work, we used more tests than necessary to predict Figure 8, in order to verify the simple mathematical model.

CONCLUSIONS

A simple mathematical method based on dimensional similarity was presented and examined by drop tests with specially constructed apparatus. The results showed good correlation between the similarity model and the experiments.

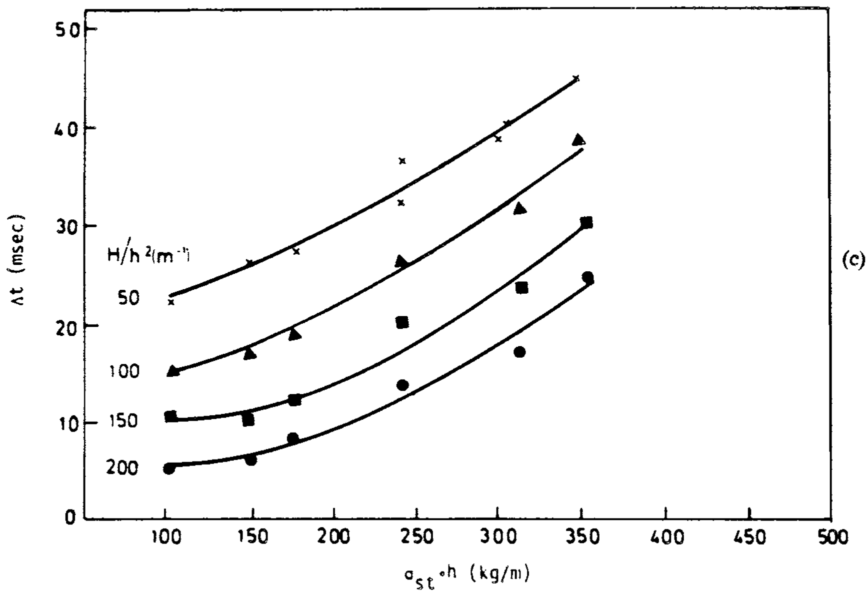

Fig. 8. (*Continued from the previous page.*)

The prediction of the similarity model is based on the assumptions that the cross-sectional area of the foam remains constant during the impact, and that the dynamic-to-static stress ratio is much higher than unity ($\sigma_{dyn}/\sigma_{st} \gg 1$). These assumptions have been proven to be true for all practical purposes in a wide range of drop tests.

It is proved that by several drop tests, one could predict the essential dynamic characteristics of the foam, that is, maximum deformation, maximum deceleration, and duration time of the impact pulse.

By using the mathematical model, the designer can predetermine the dimensions of the foam for maximum permissible deformation and deceleration for goods in impact conditions.

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